Dec-2023

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4368 **G**

Unique Paper Code : 32351101

Name of the Paper : Calculus

Name of the Course : **B.Sc. (H) Mathematics (LOCF)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any **four** *questions from Section – I.*

1. Sketch the graph of the function $f(x) = x^4 - 4x^3 + 10$ by finding intervals where it increases and decreases, relative extrema, concavity and inflection points (if any).

2. Evaluate the following limit

$$\lim_{x \to 0^+} \left(e^x - 1\right)^{1/\ln x}$$

3. Determine all the vertical and horizontal asymptotes to the curve

$$f(x) = \frac{x^2 - x - 2}{x - 3}.$$

4. A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in dollars) will be

$$C(x) = \frac{1}{8}x^2 + 4x + 200$$

and all units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that correspond to the maximum profit.

5. If $y = e^{m \sin^{-1} x}$, Show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$

SECTION - II

Attempt any **three** *questions from Section - II.*

6. Sketch the graph of the curve $r = 2 \cos 3\theta$ in polar coordinates.

7. Find an equation for a hyperbola that satisfies the condition that the curve has vertices $(0, \pm 3)$ and foci $(0, \pm 5)$.

8. Describe the graph of the equation :

$$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$$

9. Identify and sketch the curve :

$$x^2 - xy + y^2 - 2 = 0.$$

SECTION - III

Attempt any **four** *questions from Section - III.*

10. Find the arc length of the parametric curve: $x = (1 + t)^2$, $y = (1 + t)^3$ for $0 \le t \le 1$.

11. Find the area of the surface generated by revolving the curve $y = \sqrt{16 - x^2}$, $-2 \le x \le 2$ about the x-axis.

12. The region bounded by the curves $y = \sqrt{x}$, $x = 4$, $x = 9$ and $y = 0$ is revolved about the line $x = 0$. Compute the volume of the resulting solid.

13. Find the length of the catenary $y = 10 \cosh\left(\frac{x}{10}\right)$ from $x = -10$ to $x = 10$.

14. Evaluate $\int \sin^4 x \cos^2 x \, dx$.

SECTION – IV

Attempt any **four** *questions from Section – IV.*

15. Find $\lim_{t\to3}\left[t^2\hat{i}+\frac{\sin(2t-2)}{t-1}\hat{j}+e^{3t}\hat{k}\right]$.

16. If $R(t) = \ln(t^2+1)\hat{i} + (\tan^{-1} t)\hat{j} + \left(\sqrt{t^2+1}\right)\hat{k}$ is the position of a particle in space at time t. Find the angle between the velocity and acceleration vectors at time $t = 0$.

17. Determine all values of t for which the vector function $F(t) = (e^{2t}\sin 3t)\hat{i} + (t^2\cos 3t)\hat{j}$ is continuous.

18. A golf ball is hit from the tee to a green with an initial speed of 125 ft/s at an angle of elevation of 45°. How long will it take for the ball to hit the green?

19. A shell is fired from ground level with muzzle speed of 750 ft/s at an angle of 25°. An enemy gun 20,000 ft away fires a shot 2 seconds later and the shells collide 50 ft above the ground at the same speed. What is the muzzle speed and angle of elevation of the second gun?

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1591 **G**

Unique Paper Code : 2352011102

Name of the Paper : DSC-2 : Elementary Real Analysis

Name of the Course : **B.Sc. (H) Mathematics (UGCF-2022)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **three** parts from each question.
3. **All** questions carry equal marks.

1. (a) Let $a \geq 0$, $b \geq 0$ prove that $a^2 \leq b^2 \Leftrightarrow a \leq b$.

(b) Determine and sketch the set of pairs (x, y) on $\mathbb{R} \times \mathbb{R}$ satisfying the inequality $|x| \leq |y|$.

(c) Find the supremum and infimum, if they exist, of the following sets :

(i) $\left\{\sin\frac{n\pi}{2} : n \in \mathbb{N}\right\}$

(ii) $\left\{\left(\frac{1}{x} : x > 0\right)\right\}$

(d) Show that $\text{Sup}\left\{1+\frac{1}{n} : n \in \mathbb{N}\right\} = 2$.

2. (a) Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a > 0$ and let $aS = \{as : s \in S\}$. Prove that

$$\text{Sup }(aS) = a(\text{ Sup } S)$$

(b) If x and y are positive rational numbers with $x < y$, then show that there exists a rational number r such that $x < r < y$.

(c) Show that $\inf\left\{\frac{1}{n} : n \in \mathbb{N}\right\} = 0$.

(d) Show that every convergent sequence is bounded. Is the converse true? Justify.

3. (a) Using definition of limit, show that

$$\lim_{n\to\infty} \frac{n^2+3n+5}{2n^2+5n+7} = \frac{1}{2}$$

(b) Show that if $c > 0$, $\lim_{n\to\infty} (c)^{1/n} = 1$.

(c) Show that, if $x_n \geq 0$ for all n, and $\langle x_n \rangle$ is convergent then $\langle \sqrt{x_n} \rangle$ is also convergent and

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{\lim_{n\to\infty} x_n}$$

(d) Show that every increasing sequence which is bounded above is convergent.

4. (a) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all n. Prove that $\langle x_n \rangle$ is convergent and find its limit.

(b) Prove that every Cauchy sequence i convergent.

(c) Show that the sequence $\langle x_n \rangle$ defined by

$$x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}, \text{ for all } n \in \mathbb{N}$$

is convergent.

(d) Find the limit superior and limit inferior of the following sequences :

(i) $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$, for all $n \in \mathbb{N}$

(ii) $x_n = \left(1 + \frac{1}{n}\right)^{n+1}$, for all $n \in \mathbb{N}$

5. (a) Show that if a series Σa_n converges, then the sequence $\langle a_n \rangle$ converges to 0.

(b) Determine, if the following series converges, using the definition of convergence, $\sum \log\left(\frac{a_n}{a_{n+1}}\right)$ given that $a_n > 0$ for each n, $\lim_{n\to\infty} a_n = a$, $a > 0$.

(c) Find the rational number which is the sum of the series represented by the repeating decimal $0.\overline{987}$.

(d) Check the convergence of the following series :

(i) $\sum \frac{1}{2^n + n}$

(ii) $\sum \sin\left(\frac{1}{n^2}\right)$

6. (a) State the Root Test (limit form) for positive series. Using this test or otherwise, check the convergence of the following series

(i) $\sum\left(n^{1/n} - 1\right)^n$

(ii) $\sum\left(\frac{n^{n^2}}{(n+1)^{n^2}}\right)$

(b) Check the convergence of the following series :

(i) $\sum_{n=2}^{\infty}\left(\frac{1}{n \log n}\right)$

(ii) $\sum\left(\frac{n!}{n^n}\right)$

(c) Define absolute convergence of a series. Show that every absolutely convergent series is convergent. Is the converse true? Justify your answer.

(d) Check the following series for absolute or conditional convergence :

(i) $\sum(-1)^{n+1}\left(\frac{n}{n(n+3)}\right)$

(ii) $\sum(-1)^{n+1}\left(\frac{1}{n+1}\right)$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2414 **G**

Unique Paper Code : 2354001001

Name of the Paper : GE: Fundamentals of Calculus

Name of the Course : **Common Prog. Group**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory and carry equal marks.

3. This question paper has **six** questions.

4. Attempt any **two** parts from each question.

P.T.O.

1\. (a) (i) Establish that $\lim_{x \to 0} \frac{e^{1/x}-1}{e^{1/x}+1}$ does not exist.

(ii) Examine the continuity of the function

$$f(x)=\begin{cases} 0 & , \text{ if } x=0,\frac{1}{2} \\ \frac{1}{2}-x & , \text{ if } 0<x<\frac{1}{2} \\ 4x^2-1 & , \text{ if } \frac{1}{2}<x<\frac{3}{4} \\ 1-x^2 & , \text{ if } \frac{3}{4}\le x\le 1 \end{cases}$$

at $x=0, \frac{1}{2}, \frac{3}{4}$ and 1. Classify their type of discontinuities, if any.

(b) Find the derivatives of

(i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

(ii) $x^x+(\sin x)^{\log x}$.

(c) Find the n^{th} derivatives of $f(x)=\frac{1}{1-5x+6x^2}$ and $g(x) = \sin 4x \cos 2x$.

2\. (a) If $y = \tan^{-1} x$, then prove that

$$(1 + x^2)y_{n+2} + (2n + 2)xy_{n+1} + n(n + 1)y_n = 0.$$

Also find $y_n(0)$.

(b) Let $u=\cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ and $v=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$.

Prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{4}\sin 2u=0 \text{ and } x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}\sin 2v.$$

(c) Prove that if $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, then

$$\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} + \frac{\partial^2 u}{\partial^2 z} = 0.$$

3. (a) Examine the applicability of Rolle's theorem on the following functions :

(i) $f(x) = |x|$ in $[-1,1]$.

(ii) $f(x) = \tan x$ in $[0,\pi]$.

(iii) $f(x) = 10x - x^2$ in $[0,10]$.

(b) State whether the following statements are true or false. Justify your answer.

(i) Lagrange's mean value theorem is not applicable on $f(x) = \frac{1}{x-3}$ in $[5,10]$.

(ii) $\frac{x}{1+x^2} < \tan^{-1} x < x$, for all $x > 0$.

(c) State and prove Cauchy's mean value theorem. Use it to deduce Lagrange's mean value theorem.

4. (a) State Taylor's theorem. Discuss the Lagrange's and Cauchy's form of the remainder of a Taylor's series. Moreover if $f(x) = (1-x)^{5/2}$, then find the value of c as $x \to 1$ such that

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(cx).$$

(b) Find the Taylor's series for $f(x) = \log(1+x)$ where $x \in (-1,1)$ and $g(x) = e^x$.

(c) State L'Hopitals rule for form 0/0 for the limits $x \to a$, where a is a real number. Justify that whether it is applicable to compute the following limits or not.

(i) $\lim_{x \to 1} \left(\frac{2x-2}{x^3+x-2} \right)$

(ii) $\lim_{x \to 0} \left(\frac{\cos x}{x} \right)$

(iii) $\lim_{x \to 0} \left(\frac{e^{2x}-1}{\tan x} \right)$

5. (a) Determine the intervals of concavity and points of inflections of the curve $y = x^4 - 4x^3 - 18x^2 + 1$. Show that the points of inflection of the curve $y = (x - 2)\sqrt{(x-3)}$ lies on the line $3x = 10$.

(b) Find asymptotes of the curve

$$x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0.$$

(c) Determine the intervals of concavity and points of inflections of the curve $y = \frac{x^3 - x}{3x^2 + 1}$. Also, show that curve $x^2(x^2 + y^2) = a^2(x^2 - y^2)$ has no asymptotes.

6. (a) Sketch the graph of the function $y = x^2 - \frac{2}{x}$ and also identify the locations of all asymptotes, intercepts, relative extrema and inflection points.

(b) Locate the critical points and identify which critical points are stationary points for the functions

(i) $f(x) = x^3 - 3x + 2$

(ii) $g(x) = x^{1/3}(4 - x)$

(iii) $h(x) = x^2(x - 1)^{2/3}$.

(c) Trace the curve $r = 2 + 4\cos\theta$.

[This question paper contains 4 printed pages.]

Your Roll No...............

G

Sr. No. of Question Paper : 1553

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 : Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) (i) Find a cubic equation with rational coefficients having the roots

$\frac{1}{2}, \frac{1}{2}+\sqrt{2}$, stating the result used.

(ii) Find an upper limit to the roots of

$x^5 + 4x^4 - 7x^2 - 40x + 1 = 0.$ (4+3.5)

P.T.O.

(b) Find all the integral roots of

$x^4 + 4x^3 + 8x + 32 = 0.$ (7.5)

(c) Find all the rational roots of

$y^4 - \frac{40}{3}y^3 + \frac{130}{3}y^2 - 40y + 9 = 0$. (7.5)

2. (a) Express arg $(\bar{z})$ and arg $(-z)$ in terms of arg(z). Find the geometric image for the complex number z, such that $\arg(-z) \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. (2+2+3.5)

(b) Find $|z|$, arg z, Arg z, arg $\bar{z}$, arg(−z) for $z = (1 - i)(6 + 6i)$ (7.5)

(c) Find the cube roots of $z = 1 + i$ and represent them geometrically to show that they lie on a circle of radius $(2)^{1/6}$. (7.5)

3. (a) Solve $y^3 - 15y - 126 = 0$ using Cardan's method. (7.5)

(b) Let n be a natural number. Given n consecutive integers, a, a + 1, a + 2, ..., a + (n−1), show that one of them is divisible by n. (7.5)

(c) Let a and b be two integers such that gcd(a, b) = g. Show that there exists integers m and n such that g = ma + nb. (7.5)

4. (a) Let a be an integer such that a is not divisible by 7. Show that $a \equiv 5^k \pmod 7$ for some integer k. (7.5)

(b) Let a and b be two integers such that 3 divides $(a^2 + b^2)$. Show that 3 divides a and b both. (7.5)

(c) Solve the following pair of congruences, if possible. If no solution exists, explain why? (7.5)

$x + 5y \equiv 3 \pmod 9$

$4x + 5y \equiv 1 \pmod 9$

5. (a) Consider a square with four corners labelled as follows :

Describe the following motions graphically:

(i) R_0 = Rotation of 0 degree.

(ii) R_{90} = Rotation of 90 degrees counterclockwise.

(iii) R_{180} = Rotation of 180 degrees counterclockwise.

(iv) R_{270} = Rotation of 270 degrees counterclockwise.

(v) H = Flip about horizontal axis.

(vi) V = Flip about vertical axis.

(vii) D = Flip about the main diagonal.

(viii) D1 = Flip about the other diagonal.

Identify the motion that can act as identity under the composition of two motions. Further, find out the inverse of each motion. (3.5+1+3)

(b) Show that the set $G = \{f_1, f_2, f_3, f_4\}$, is a group under the composition of functions defined as, $fog(x) = f(g(x))$ for f, g in G, where $f_1(x) = x$, $f_2(x) = -x$, $f_3(x) = 1/x$, $f_4(x) = -1/x$ for all non-zero real number x. (7.5)

(c) Define the inverse of an element in a group G. Show that $(a.b)^{-1} = b^{-1}a^{-1}$ for all a, b in G. Further show that if $(a.b)^{-1} = a^{-1}.b^{-1}$ for all a, b in G, then G is Abelian. (4+3.5)

6. (a) Define Z(G), the center of a group G. Show that Z(G) is a subgroup of G. (2+5.5)

(b) Define order of an element a in group G. Further show that if order of a is n, and $a^m = e$, where m is an integer, then n divides m. (2+5.5)

(c) Find the generators of the cyclic group Z_{30}. Further describe all the subgroups of Z_{30} and find the generators of the subgroup of order 15 in Z_{30}. (2+3.5+2)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1629 **G**

Unique Paper Code : 2352011103

Name of the Paper : DSC-3: Probability and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.
5. Use of non-programmable scientific calculators and statistical tables is permitted.

P.T.O.

1. (a) The following table gives the accompanying specific gravity values for various wood types used in construction. Construct a stem and leaf display and comment on any interesting features of the display

.31	.35	.36	.36	.37	.38	.40	.40	.40
.41	.41	.42	.42	.42	.42	.42	.43	.44
.45	.46	.46	.47	.48	.48	.48	.51	.54
.54	.55	.58	.62	.66	.66	.67	.68	.75

(b) The following data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine. Compute the Median, Upper Fourth (third quartile) and Lower Fourth (first quartile)

1.6	2.0	2.6	3.0	3.9	3.5	4.5	4.6	4.8	5.0

(c) The following table gives the data on oxidation-induction time (measured in minutes) for various commercial oils.

87	103	130	160	180	195	132	145	211	105

(i) Calculate the sample variance and standard deviation.

(ii) If the observations were re-expressed in hours, what would be the resulting values of the sample variance and sample standard deviation? Answer without reperforming the calculations.

2. (a) If A and B are any two events, then show that $P(A \cap B') = P(A) - P(A \cap B)$. Hence or otherwise prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will not be discovered?

(c) State Baye's Theorem A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two- night visit. In addition, 10% of day visitors ultimately make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit?

3. (a) In a group of five potential blood donors a, b, c, d, and e, only a and b have Opositive (O+) blood type. Five blood samples, one from each individual, will be typed in random order until an O+ individual is identified. Let the random variable Y = the number of typings necessary to identify an O+ individual.

(i) Find the probability mass function (pmf) of Y.

(ii) Draw the line graph and probability histogram of the pmf.

(b) The n candidates for a job have been ranked 1,2,3, ... n. Each candidate has an equal chance of being selected for the job. Let the random variable X be defined as

X = the rank of a randomly selected candidate

(i) Find the probability mass function (pmf) of X.

(ii) Compute E(X) and V(X).

(c) For any random variable X, prove that $V(aX + b) = a^2V(X)$ and $\sigma_{aX+b} = |a|\sigma_X$.

4. (a) The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with probability density function (pdf)

$$f(x) = \begin{cases} \frac{3}{2}(1-x^2) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

(i) the cumulative density function (cdf) of sales

(ii) E(X)

(iii) V(X)

(iv) σ_X

(b) The reaction time for an in-traffic response to a brake signal from standard brake lights can be modelled with a normal distribution having mean value 1.25 sec and standard deviation of 0.46 sec.

What is the probability that the reaction time is between 1.00 sec and 1.75 sec? If 2 sec is a critical long reaction time, what is the probability that actual reaction time will exceed this value?

(c) If X is a binomially distributed random variable with parameters n and p, prove that

(i) $E[X] = np$

(ii) $V[X] = np(1 - p)$

5. (a) If 75% of all purchases in a certain store are made with a credit card and the random variable, X = number among ten randomly selected purchases made with a credit card is a Binomial variate, then determine

(i) E(X)

(ii) V(X)

(iii) σ_X

(iv) The probability that X is within 1 standard deviation of its mean value.

(b) Let X denote the amount of time a book on two-hour reserve is actually checked out, and suppose the cumulative density function is

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

(i) Calculate $P(.5 \le X \le 1)$.

(ii) What is the median checkout duration $\tilde{\mu}$?

(iii) Obtain the density function f(x).

(c) Theamount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size c will ensure that overflow occurs only 0.5% of the time?

6. (a) Toughness and fibrousness of asparagus are major determinants of quality. This was reported in a study with the following data on x = shear force (kg) and y = percent fiber dry weight.

X	46	48	55	57	60	72	81	85	94	109
y	2.18	2.10	2.13	2.28	2.34	2.53	2.28	2.62	2.63	2.50

(i) Calculate the value of the sample correlation coefficient. Based on this value, how would you describe the nature of relationship between the two variables?

(ii) If shear force is expressed in pounds, what happens to the value of r? Why?

(b) An experiment was performed to investigate how the behavior of mozzarella cheese varied with temperature. The following observations on x = temperature. The following observations on x = Temperature and y = elongation(%) at failure of the cheese.

X	59	63	68	72	74	78	83
y	118	182	247	208	197	135	132

(i) Determine the equation of the estimated regression line using the principle of least square.

(ii) Estimate the elongation at failure of the cheese when the temperature is 70.

(c) The inside diameter of a randomly selected piston ring is a random variable with mean value of 12 cm and standard deviation 0.04 cm. If $\bar{X}$ is the sample mean diameter for a random sample of n = 16 rings,

(i) where is the sampling distribution of $\bar{X}$ centered,

(ii) what is the standard deviation of the $\bar{X}$ distribution.

(iii) How likely is it that the sample mean diameter exceeds 12.01?

(b) An experiment was performed to investigate how the behavior of mozzarella cheese varied with temperature. The following observations on x = temperature and y = elongation(%) at failure of the cheese.

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(ii) what is the standard deviation of the $\bar{X}$ distribution.

(iii) How likely is it that the sample mean diameter exceeds 12.01?

(b) If $y = \tan^{-1} x$, prove that

$$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$

(c) State Euler's theorem and if $z = \sec^{-1}\frac{x^3+y^3}{x+y}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z$.

2. (a) Let $f(x) = |x - 5|$, show that f is continuous but not differentiable at $x = 5$.

(b) Find n^{th} derivative of

(i) $\frac{1}{1-5x+6x^2}$

(ii) $\sin 3x \sin 2x$

(c) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, prove that

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}.$$

3. (a) State Rolle's theorem. Show that there is no real no. k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0,1].

(b) Verify Lagrange's Mean Value Theorem for the following functions :

(i) $f(x) = \sqrt{x^2 - 4}, \quad x \in [2,4]$

(ii) $f(x) = x(x-1)(x-2), \quad x \in \left[0, \frac{1}{2}\right]$

(c) Determine the values of a and b for which.

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3}$$ exists and equals 1.

4. (a) State Maclaurin's theorem. Also, find the Maclaurin's series for

$f(x) = \log(1 + x), \ x \in (-1, 1]$.

(b) State Cauchy's mean value theorem. Verify it for the following functions :

(i) $f(x) = x^2, \ g(x) = x$ in $[-1,1]$,

(ii) $f(x) = \frac{1}{x^2}, \ g(x) = \frac{1}{x}$ in [2,3].

(c) Use Lagrange's Mean Value theorem to prove that

$$\frac{x}{1+x^2} < \tan^{-1}x < x, \ x > 0.$$

5. (a) Find all the asymptotes of the curve

$$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0.$$

(b) Trace the curve

$$x^2(a^2 - x^2) = a^2y^2, \ a > 0.$$

(c) If $u_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$, show that $u_n = \frac{n-1}{n} u_{n-2}$. Hence evaluate u_5.

6. (a) Prove that the curve

$$(a + y)^2(b^2 - y^2) = x^2y^2, \ a > 0, \ b > 0$$

has at $x = 0$, $y = -a$, a node if $b > a$, a cusp if $b = a$ and a conjugate point if $b < a$.

(b) Trace the curve

$$x(x - 3a)^2 = 9ay^2, \ a > 0.$$

(c) Determine the intervals of concavity and points of inflexion of the curve

$$y = 3x^5 - 40x^3 + 3x - 20.$$

[This question paper contains 4 printed pages.]

Your Roll No..............

G

Sr. No. of Question Paper : 4801

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : **B.Sc. (Prog.)**

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.

Unit I

1. (a) Find the inverse of $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ in $GL(2, Z_5)$, the group of 2×2 non-singular matrices over Z. Verify the answer by direct calculation. (6)

P.T.O.

(b) Describe the group of symmetries of a square and draw its Cayley's table. (6)

(c) Find the orders of each of the elements of U(14). Show that it is cyclic and find all its generators. (6)

2. (a) Let G be a group and let $a \in G$. Prove that $\langle a^{-1} \rangle = \langle a \rangle$. (6)

(b) State and prove Lagrange's Theorem. (6)

(c) If G is a group such that $x^2 = e$ for all elements x of G where e is the identity element of G then prove that G is an abelian group. (6)

3. (a) Prove that in a finite group, the order of each element of the group divides the order of the group. (6)

(b) Define an alternating group. Find all the elements of A_4. (6)

(c) Let $\sigma = (1,5,7)(2,5,3)(1,6)$. Then find σ^{17}. (6)

Unit II

4. (a) (i) Let a belong to a ring R. Let $S = \{x \in R : ax = 0\}$. Show that S is a subring of R.

(ii) Prove or disprove $3Z \cup 5Z$ is a subring of the ring Z of integers. (6.5)

(b) State the Subring Test and Show that the set $S = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} : x \in R \right\}$ is subring of the ring of all 3×3 matrices over real numbers. Also find the unity of S. (6.5)

(c) Define an ideal of a ring R. Prove that the intersection of two ideals of a ring R is an ideal of a ring R. What can you say about the union of two ideals of a ring R? Justify. (6.5)

Unit III

5. (a) Prove that a non - empty subset W of a vector space V(F) is a subspace of V if and only if $\alpha x + \beta y \in W$ for all $\alpha, \beta \in F$ and $x, y \in W$. (6.5)

(b) Let {a, b, c} be a basis for the Vector Space. Prove that the set {a+b, b+c, c+a}, {a, a+b, a+b+c} are also bases of R^3. (6.5)

(c) Define Linear Transformation. Check whether the mapping T: $R^2 \rightarrow R^2$ defined by $T(x, y) = (1 + x, y)$ is a Linear Transformation. (6.5)

6. (a) Suppose that T: $R^2 \rightarrow R^2$ is a Linear Transformation. If T (1,0) = (1,4) and T (1,1) = (2,5). Find T (2,3). Is T one-to-one? (6.5)

(b) Let T: V -> U be a Linear Transformation. Then prove that T is one-to-one if and only if the null space N(T) = {0}. (6.5)

(c) Let T: $R^2 \rightarrow R^3$ be a Linear Transformation defined by T (x,y) = (x, x+y, y). Find the Range, Rank, Kernal and Nullity of T. (6.5)

(1000)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1763 **G**

Unique Paper Code : 2352572301

Name of the Paper : Differential Equations

Name of the Course : **B.Sc. (Physical Science and Mathematical Science) with Operational Research and Bachelor of Arts**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts form each question.
3. **All** questions carry equal marks.

P.T.O.

1. (a) Show that every function f defined by

$$f(x) = (x^3 + c)e^{-3x},$$

where c is an arbitrary constant, is a solution of the differential equation $\frac{dy}{dx} + 3y = 3x^2e^{-3x}$. Also determine whether the equation

$$\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + xy = xe^x$$

is linear or nonlinear. (7½)

(b) Write the definition of exact differential equation and in the following equation determine the constant A such that the equation is exact, and solve the resulting exact equation:

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0$$ (7½)

(c) Solve the initial value problem that consists of the differential equation

$$x \sin y \, dx + (x^2 + 1) \cos y \, dy = 0$$

and the initial condition $y(1) = \pi$ (7½)

2. (a) The human population of a certain island satisfies the logistic law

$$\frac{dx}{dt} = \frac{1}{100x} - \frac{1}{(10)^8}x^2$$ with $k = 0.03$, $\lambda = 3(10)^{-8}$,

and time t measured in years.

(i) If the population in 1980 is 200,000, find the formula for the population in the future Years.

(ii) What will be the formula for the population in the year 2000. (7½)

(b) Find the value of K such that the parabolas $y = c_1x^2 + K$ are the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 - y = c_2$. (7½)

(c) Solve the initial value problem

$$x\frac{dy}{dx} + y = (xy)^{\frac{3}{2}}, \quad y(1) = 4.$$ (7½)

3. (a) Given that x, x^2 and x^4 are all solutions of the equation

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0.$$

Show that they are linearly independent on the interval $0 < x < \infty$ and write the general solution. (7½)

(b) Prove that if $f_1(x)$ and $f_2(x)$ are two solution of

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0,$$ then

$cf_1(x) + c_2f_2(x)$ is also a solution of this equation, where c_1 and c_2 are arbitrary constant. (7½)

(c) The roots of the auxiliary equation, corresponding to a certain 12^{th} order homogeneous linear differential equation with constant coefficients, are

2, 2, 2, 2, 2, 2, 3 + 4i, 3 − 4i, 3 + 4i, 3 − 4i, 3 + 4i, 3 − 4i.

Write the general solution and also find the general solution of $\frac{d^2y}{dx^2} + y = 0$. (7½)

4. (a) Find the general solution of the differential equation :

$$4\frac{d^2y}{dx^2}-12\frac{dy}{dx}+9y=0,\ y(0)=4 \text{ and } y'(0)=9. \quad (7½)$$

(b) Use the method of undetermined coefficients, find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=\cos 4x. \quad (7½)$$

(c) Use the method of variation of parameter to find the general solution of the differential equation :

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=e^x \tan 2x. \quad (7½)$$

5. (a) Find the general solution of Cauchy problem for first order PDE.

$$x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=2xy \text{ with } u=2 \text{ on } y=x^2 \quad (7½)$$

(b) Find the Solution of characteristic equation for the first order PDE.

$$\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u+1 \text{ with } u(x, y)=x^2 \text{ on } y=x^2$$

(7½)

(c) Find the general solution of the equation :

$$(y-ux)p+(x+yu)q=x^2+y^2 \quad (7½)$$

6. (a) Find the general solution of Cauchy problem for first order PDE.

$$\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=y \text{ with } u(0, y)=y^2 \quad (7½)$$

(b) Obtain the general solution of the equation.

$xu_x + yu_y = xe^{-u}$ with Cauchy data $u = 0$ on $y = x^2$ (7½)

(c) Reduce the equation: $y^2u_{xx} + 3yu_{xy} + 3u_x = 0$, $y \neq 0$ find the general solution. (7½)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2253 **G**

Unique Paper Code : 2354002001

Name of the Paper : DIFFERENTIAL EQUATIONS

Name of the Course : **COMMON PROG GROUPS (Generic Elective)**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Determine the values of p for which the function g defined by $g(x) = x^p$ is a solution of the differential equation

$$x^3\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}-10x\frac{dy}{dx}-8y=0. \qquad (7.5)$$

(b) Solve the equation

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0. \qquad (7.5)$$

(c) Solve the differential equation

$$2r(s^2 + 1)dr + (r^4 + 1)ds = 0. \qquad (7.5)$$

2. (a) Solve the differential equation

$$\frac{dx}{dt}+\frac{x}{t^2}=\frac{1}{t^2}. \qquad (7.5)$$

(b) Solve the differential equation

$$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0,$$

by first finding an integrating factor. (7.5)

(c) Determine the value of K such that the parabolas $y = c_1x^2 + K$ are the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 - y = c_2$. (7.5)

3. (a) Find the solution of the differential equation

$$4\frac{d^2y}{dx^2}+4\frac{dy}{dx}+37y=0,\ y(0)=2,\ y'(0)=-4. \qquad (7.5)$$

(b) Find the general solution of the differential equation

$$y'' - 5y' + 6y = 4e^{2x},$$

using method of undetermined coefficients. (7.5)

(c) Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+y=\sec^2 x. \qquad (7.5)$$

4. (a) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=x^3. \qquad (7.5)$$

(b) Solve the linear system

$$\frac{dx}{dt}+\frac{dy}{dt}+2y=\sin t,\quad \frac{dx}{dt}+\frac{dy}{dt}-x-y=0 \qquad (7.5)$$

(c) Show that $x = 2e^{2t}$, $y = -3e^{2t}$ and $x = e^{7t}$, $y = e^{7t}$ are two linearly independent solutions on every interval $a \le t \le b$ of the homogeneous linear system

$$\frac{dx}{dt}=5x+2y,\quad \frac{dy}{dt}=3x+4y$$

Write the general solution. (7.5)

5. (a) Apply $\sqrt{u} = v$ and $v(x, y) = f(x) + g(y)$ to solve

$$x^4 u_x^2 + y^2 u_y^2 = 4u. \qquad (7.5)$$

(b) Find the solution of the initial-value systems

$$u_t + u u_x = e^{-t}v, \; v_t - av_x = 0,$$

with $u(x, 0) = x$ and $v(x, 0) = e^x$. (7.5)

(c) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2. \qquad (7.5)$$

6. (a) Given that the parabolic equation

$$u_{xx} = au_t + bu_x + cu + f,$$

where the coefficients are constants, by the substitution $u = v\,e^{bx/2}$ and for the case $c = -(b^2/4)$, show that the given equation is reduced to the heat equation

$$v_{xx} = a v_t + g,$$

where $g = f e^{-bx/2}$. (7.5)

(b) Find the solution of the Cauchy problem

$$x u_x - y u_y + y^2 u = y^2. \qquad (7.5)$$

(c) Apply $v = \ln u$ and then $v(x, y) = f(x) + g(y)$ to solve

$$x^2 u_x^2 + y^2 u_y^2 = (x y u)^2. \qquad (7.5)$$

[This question paper contains 4 printed pages.]

Your Roll No................

G

Sr. No. of Question Paper : 3924

Unique Paper Code : 32355301

Name of the Paper : GE – III Differential Equations

Name of the Course : **Generic Elective / Other than B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Solve the following differential equations :

$$(4x + 3y + 1)dx + (x + y + 1)dy = 0, \; y(3) = -4.$$ (6.5)

(b) Solve the Initial Value problem :

$$x^2 \frac{dy}{dx} + xy = \frac{y^3}{x}, \quad y(1) = 1$$ (6.5)

P.T.O.

(c) Find the orthogonal trajectories of the family of ellipse having centre at the origin , a focus at the point (c,0) and semimajor axis of length 2c. (6.5)

2. (a) (i) Find the Wronskian of the set {1 − x, 1 + x, 1 − 3x} and hence find their linear independence or dependence on $(-\infty,\infty)$. (6)

(ii) Solve the differential equation :

$$\frac{dy}{dx}=\frac{2+ye^{xy}}{2y-xe^{xy}}.$$

(b) Given that y = x is a solution of the differential equation

$$\left(x^2-x+1\right)\frac{d^2y}{dx^2}-\left(x^2+x\right)\frac{dy}{dx}+(x+1)y=0,$$

find a linearly independent solution by reducing the order. Write the general solution also. (6)

(c) Solve the following differential equations :

(i) $x\frac{dy}{dx}-2y=2x^4,\ y(2)=8$

(ii) $(2xy^2+y)dx+(2y^3-x)dy=0.$ (6)

3. (a) Solve the Initial value Problem

$$\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}-10\frac{dy}{dx}=0, \quad y(0)=7, \quad \frac{dy}{dx}(0)=0,$$
$$\frac{d^2y}{dx^2}(0)=70.$$ (6.5)

(b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=8\sin(3x).$$ (6.5)

(c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2}+y=\operatorname{cosec}(x).$$ (6.5)

4. (a) Given that sin x is a solution of the differential equation

$$\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$$

find the general solution. (6)

(b) Find the general solution of the differential equation by assuming x > 0,

$$x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=0$$ (6)

(c) Find the general solution of the given linear system

$$2\frac{dx}{dt}+\frac{dy}{dt}-x-y=e^{-t}, \quad \frac{dx}{dt}+\frac{dy}{dt}+2x+y=e^{t}.$$ (6)

5. (a) Find the solution of the linear partial differential equation

$(y-u)u_x + (u-x)u_y = x-y$, with Cauchy data $u=0$ on $xy=1$. (6)

(b) Find the solution of the following partial differential equation by the method of separation of variables

$u_x + u = u_y$, $u(x, 0) = 4\exp(-3y)$. (6)

(c) Reduce the equation to canonical form and obtain the general solution

$$u_x + 2xy\, u_y = x. \quad (6)$$

6. (a) Find the general solution of the linear partial differential equation

$yz\, u_x - xz\, u_y + xy(x^2 + y^2)\, u_z = 0$. (6.5)

(b) Reduce the equation

$u_{xx} - \frac{1}{c^2} u_{yy} = 0$, $c \neq 0$ where c is a constant, into canonical form and hence find the general solution. (6.5)

(c) Reduce the following partial differential equation with constant coefficients,

$$3\, u_{xx} + 10\, u_{xy} + 3\, u_{yy} = 0$$

into canonical form and hence find the general solution. (6.5)

(1000)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1610 **G**

Unique Paper Code : 2352012303

Name of the Paper : Discrete Mathematics

Name of the Course : **B.Sc. (H) – DSC**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

P.T.O.

1. (a) (i) Define covering relation in an ordered set and finite ordered set. Prove that if X is any set, then the ordered set $\wp(X)$ equipped with the set inclusion relation given by $A \leq B$ iff $A \subseteq B$ for all $A, B \in \wp(X)$, a subset B of X covers a subset A of X iff $B = A \cup \{b\}$ for some $b \in X - A$.

(ii) State Zorn's Lemma.

(b) (i) Give an example of an ordered set (with diagram) with more than one maximal element but no greatest element. Specify maximal elements also.

(ii) Define when two sets have the same cardinality. Show that

- $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$
- $\mathbb{Z}$ and $2\mathbb{Z}$

have the same cardinality.

(c) Let $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n. Draw Hasse diagram for the subset $S = \{1,2,4,5,6,12,20,30,60\}$ of $(\mathbb{N}_0, \leq)$. Find elements $a, b, c, d \in S$ such that $a \vee b$ and $c \wedge d$ does not exist in S.

2. (a) Define an order preserving map. In which of the following cases is the map $\varphi : P \to Q$ order preserving?

(i) $P = Q = (\mathbb{N}_0, \leq)$ and $\varphi(x) = nx$ ($n \in \mathbb{N}_0$ is fixed).

(ii) $P = Q = (\wp(\mathbb{N}), \subseteq)$ and φ defined by

$$\varphi(U) = \begin{cases} \{1\}, & 1 \in U \\ \{2\}, & 2 \in U \text{ and } 1 \notin U, \\ \emptyset, & \text{otherwise} \end{cases}$$

where $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ iff m divides n and $\wp(\mathbb{N})$ be the power set of $\mathbb{N}$ equipped with the partial order given by $A \leq B$ iff $A \subseteq B$ for all $A, B \in \wp(\mathbb{N})$.

(b) For disjoint ordered sets P and Q define order relation on $P \cup Q$. Draw the diagram of ordered sets (i) 2×2 (ii) $3 \cup \bar{3}$ (iii) $M_2 \oplus M_3$ where $M_n = 1 \oplus \bar{n} \oplus 1$.

(c) Let $X = \{1,2,..., n\}$ and define $\varphi: \wp(X) \to 2^n$ by $\varphi(A) = (\varepsilon_1,..., \varepsilon_n)$ where

$$\varepsilon_i = \begin{cases} 1 & \text{if } i \in A \\ 0 & \text{if } i \notin A \end{cases}$$

Show that φ is an order-isomorphism.

3. (a) Let L and K be lattices and $f: L \to K$ a lattice homomorphism.

(i) Show that if $M \in \text{Sub } L$, then $f(M) \in \text{Sub } K$.

(ii) Show that if $N \in \text{Sub } K$, then $f^{-1}(N) \in \text{Sub}_0 L$, where $\text{Sub}_0 L = \text{Sub } L \cup \emptyset$.

(b) Let L be a lattice.

(i) Assume that $b \leq a \leq b \vee c$ for $a, b, c \in L$. Show that $a \vee c = b \vee c$.

(ii) Show that the operations $\vee$ and $\wedge$ are isotone in L, i.e. $b \leq c \Rightarrow a \wedge b \leq a \wedge c$ and $a \vee b \leq a \vee c$.

(c) Let L and M be lattices. Show that the product $L \times M$ is a lattice under the operations $\vee$ and $\wedge$ defined as

$$(x_1,y_1) \vee (x_2,y_2) := (x_1 \vee x_2, y_1 \vee y_2),$$

$$(x_1,y_1) \wedge (x_2,y_2) := (x_1 \wedge x_2, y_1 \wedge y_2)$$

4. (a) Let L be a distributive lattice. Show that $\forall x, y, z \in$ L, the following laws are equivalent :

(i) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

(ii) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

(b) Define modular lattices. Show that every distributive lattice is modular. Is the converse true? Give arguments in support of your answer.

(c) (i) Prove that for any two elements x, y in a lattice L, the interval

$[x,y] := \{a \in L | x \leq a \leq y\}$ is a sublattice of L.

(ii) Let f be a monomorphism from a lattice L into a lattice M. Show that L is isomorphic to a sublattice of M.

5. (a) (i) Prove that $(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$ for all x, y in a Boolean algebra.

Deduce that $x \leq y \Leftrightarrow x' \geq y'$ for all $x, y \in B$.

(ii) Show that the lattice $B = (\{1,2,3,6,9,18\}, \text{gcd}, \text{lcm})$ of all positive divisors of 18 does not form a Boolean algebra.

(b) Find the conjunctive normal form of

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'$$

(c) Use a Karnaugh Diagram to simplify

$$p = x_1x_2x_3 + x_2x_3x_4 + x_1'x_2x_4' + x_1'x_2x_3x_4' + x_1'x_2x_4'$$

6. (a) Use the Quine-McCluskey method to find the minimal form of

$$wxyz' + wxy'z' + wx'yz + wx'yz' + w'x'yz + w'x'yz' + w'x'y'z$$

(b) Draw the contact diagram and determine the symbolic representation of the circuit given by

$$p = x_1x_2(x_3+x_4) + x_1x_2(x_5+x_6)$$

(c) Give mathematical models for the following random experiments

(i) when in tossing a die, all outcomes and all combinations are of interest.

(ii) when tossing a die, we are only interested whether the points are less than 3 or greater than or equal to 3.

(21)

[This question paper contains 8 printed pages.]

Your Roll No................

Sr. No. of Question Paper : 1534 **G**

Unique Paper Code : 2352012301

Name of the Paper : Group Theory

Name of the Course : **B.Sc. (H) Mathematics – DSC**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator is not allowed.

P.T.O.

1. (a) Derive a formula for finding the order of a permutation of a finite set written in disjoint cycle form. Let $\beta = (1,3,5,7,9)\ (2,4,6)\ (8,10)$. What is the smallest positive integer for which $\beta^m = \beta^{-5}$?

(b) Let $\alpha, \beta \in S_n$. Prove that $\alpha\beta$ is even if and only if either both α and β are even or both α and β are odd.

(c) Suppose H is a subgroup of S_n of odd order. Prov that H is a subgroup of A_n.

2. (a) Let H and K be normal subgroups of a group such that $H \cap K = \{e\}$, then prove that th elements of H and K commute. Give an examp of a non-Abelian group whose all subgroups normal.

(b) Let G be group and suppose that N is a normal subgroup of G and H is any subgroup of G. Prove that $N \cap H$ is a normal subgroup of G. Justify this statement with an example too.

(c) Suppose H and K are subgroups of a group G. If aH is a subset of bH, then prove that H is a subset of K. Is the converse true except when a and b are identity? Justify your answer with an example.

3. (a) State and prove first isomorphism theorem. Show that $\mathbb{Z}/n\mathbb{Z}$ is isomorphic to $\mathbb{Z}_n$, for all $n \in \mathbb{N}$.

(b) Let ϕ be an isomorphism from a group G onto a group G′, prove that

(i) G is cyclic if and only if G′ is cyclic.

(ii) for all elements $g \in G$, the order of g is equal to order of $\phi(g)$.

(c) (i) Let G be a group, prove that the mapping ϕ defined as

$$\phi(g) = g^{-1} \text{ for all } g \in G$$

is an automorphism if and only if G is an Abelian group.

(ii) Suppose ϕ is a homomorphism from a group G onto a group G′, prove that

$$\phi(Z(G)) \subseteq Z(G').$$

Here, Z(G) denotes the center of G.

4. (a) (i) Suppose that G is a finite Abelian group and G has no element of order 2. Show that the mapping ϕ defined as

$$\phi(g) = g^2 \text{ for all } g \in G$$

is an automorphism. Is this mapping ϕ an automorphism when G is an infinite group and has no element of order 2? Justify your answer.

(ii) Prove that a homomorphism ϕ from a group G onto a group G′ is one-one if and only if $\ker \phi = \{e\}$, where e is the identity element of G.

(b) (i) Suppose ϕ is a homomorphism from a group G to a group G′ and $g \in G$ such that $\phi(g) = g'$, prove that

$$\phi^{-1}(g') = g \ker \phi.$$

(ii) For $\mathbb{C}^*$, the multiplicative group of non-z complex numbers, prove that the mappin defined as

$$\phi(z) = z^5 \text{ for all } z \in \mathbb{C}^*$$

is a homomorphism. Also, find the kerne ϕ.

(c) (i) Prove that every normal subgroup c group G is the kernel of a homomorphisn G.

(ii) Suppose that ϕ is a homomorphism f U(40) to U(40) and its kernel is give ker ϕ = {1,9,17,33}. If $\phi(11)$ = 11, all elements of U(40) that are mappe 11.

5. (a) Find all subgroups of order 3 in $\mathbb{Z}_9 \oplus \mathbb{Z}_3$.

(b) Let G and H be finite cyclic groups. Then G ⊕ H is cyclic if and only if |G| and |H| are relatively prime.

(c) Using the concept of external direct product, determine the last two digits of the number $23^{(23)}$.

6. (a) Determine the number of elements of order 15 and the cyclic subgroups of order 15 in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$.

(b) Define the internal direct product of n normal subgroups of a group. If a group G is the internal direct product of a finite number of normal subgroups

$H_1, H_2, \ldots, H_n$, then show that G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$.

(c) Let G = {1,8,12,14,18,21,27,31,34,38,44,47,51,53, 57,64} under the operation multiplication modulo 65. Determine the isomorphism class of the group G.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4404 **G**

Unique Paper Code : 32351302

Name of the Paper : Group Theory – I

Name of the Course : **B.Sc. (Hons) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question from **Q2** to **Q6**.
4. In the question paper, given notations have their usual meaning unless until stated otherwise.

P.T.O.

1. Give **short** answers to the following questions. Attempt any **six**.

(i) Find an element X in D_4 such that $R_{90}VXH = D'$.

Where R_{90} = Rotation of 90°, V = Flip about a vertical axis, H = Flip about a horizontal axis, D' = Flip about the other diagonal.

(ii) Is G = {1,2,3,4,5} a group under multiplication modulo 6? In general when is G = {1,2,...,n-1}; $n \geq 2$, a group under multiplication modulo n? Answer both in a few lines.

(iii) Can a non-Abelian group have a non-trivial Abelian subgroup? Give short answer in few lines.

(iv) Let G be a group such that $x = x^{-1}$, for all $x \in G$. Prove that G is Abelian.

(v) Give an example of a non-cyclic group, whose every proper subgroup is cyclic.

(vi) Prove that a group of order 4 is Abelian.

(vii) List all the generators of (Z, +), Z_7 and Z_8.

(viii) For any integer n > 2, show that there are at least two elements in U(n) that satisfy $x^2 = 1$.

(6×2=12)

2. (a) Prove that

$$G = \left\{ \begin{bmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{bmatrix} : a \in \mathbb{R} \right\}$$

is an infinite Abelian group under matrix multiplication.

(b) Define a cyclic subgroup of a group. Is it Abelian or Non-Abelian? Justify your answer. Prove that

$$H = \left\{\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} : n \in Z\right\}$$

is a cyclic subgroup of GL(2, ℝ).

(c) Prove that the subgroup of a cyclic group is cyclic. Find the smallest subgroup of (Z, +) containing 8 and 14. (2×6.5=13)

3. (a) Prove that the order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. (6)

(b) (i) Show that S_7 has an element of order 12. Find one such element.

(ii) Give two reasons why the set of odd permutations in S_n is not a subgroup. (3+3=6)

(c) (i) Let G = U (24), H = {1,7}. Write all the distinct left cosets of H in G.

(ii) Prove that a group of order 98 can have at the most one subgroup of order 49. (3+3=6)

4. (a) (i) Let H be a subgroup of G and a and b belongs to G. Then prove that

$aH = bH$ iff $a^{-1}b \in H$

(ii) State Lagrange's Theorem for finite groups and prove that every group of prime order is cyclic. (3+3.5=6.5)

(b) (i) Let H = {I, (12)(34)}, G = A_4. Show that H is not a normal subgroup of G.

(ii) Is the order of a factor group of an infinite group is infinite? Give example or counter example to support your answer.

(3+3.5=6.5)

(c) (i) Prove that Z (G), the centre of a group G, is always a normal subgroup of G.

(ii) Let G = Z, the group of integers under addition. Write all the elements of factor group Z/20Z of Z. Is this factor group cyclic? Give explanation in support of your answer.

(3+3.5=6.5)

5. (a) If H is a subgroup of a group G and K is a normal subgroup of G, then prove that H/(H ∩ K) is isomorphic to HK/K.

(b) Determine the possible homomorphisms from Z_{20} to Z_{10}. Also, find which of the homomorphisms are onto.

(c) Prove or disprove the following by justifying them :

(i) U(8) ≈ Q_8, the group of Quaternions.

(ii) U(20) ≈ D_4

(iii) (Q, +) ≈ (Z, +)

(2×6=12)

6. (a) If ϕ is an isomorphism from a group G onto a group $\bar{G}$, then prove that

$|\phi(g)| = |g|$ for all $g \in G$.

(b) Let $\mathbb{C}$ be the set of complex numbers and

$M = \left\{\begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in \mathbb{R}\right\}$. Prove that $\mathbb{C}$ and M are isomorphic under addition and that $\mathbb{C}^*$ and M^*, the non-zero elements of M, are isomorphic under multiplication.

(c) Suppose that ϕ is a homorphism from U/(30) to U(30) and that Kerϕ = {1,11}. If $\phi(7) = 7$, find all the elements of U(30) that are mapped to 7. State and prove the result used.

(2×6.5=13)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4274 **G**

Unique Paper Code : 32353301

Name of the Paper : SEC: LaTeX and HTML

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.

1. Fill in the blanks: (4×½=2)

   (i) The symbol ∞ can be produced in LaTeX using the command ______ .

   (ii) The ______ produces a line of length 40 in the direction given by the vector (0,1).

P.T.O.

(iii) _______ tag is used for separating a line of te
in the HTML.

(iv) The first command after the preamble _______
generates the title page in beamer.

2. Attempt any **eight** parts : (8×2=16

(i) Correct the following input as per LaTe commands and write its output If $$\Theta=n\pi then $$ sin n\pi=0$ for all $$n=0,1,2,3 \ldots$

(ii) Write the input command in LaTeX to produc the following :

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \ \& \ B=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(iii) Write the LaTeX commands to draw a rectangle using the picture environment.

(iv) Write the command to include the figure "myfig.eps" in a LaTeX document.

(v) Give the LaTeX command to draw a sector of a circle.

(vi) Write the following postfix expressions in standard form :

e x exp 1 x 2 exp div x 2 exp add mul

(vii) Write the HTML code to put an image and hyperlink with an example.

(viii) What does the <title>....</title> section of a Web page contain? Where does the resulting text appear?

(ix) Correct the following input of beamer commands and write output

```
\documentclass {Beamer}
\title{My Topic}
\author{XYZ}
\institute{University of Delhi}
\begin{Frame}
\titlepage
\end{Frame}
```

```
\begin{document}
\begin{Frame}
\Huge{Thank You}
\end{Frame}
\end (document)
```

(x) Correct the following input as per HTML commands

<p> This is <bf><it> bold and italics <\bf><\it><\p>

3. Attempt any **four** parts : (4×5=20)

(i) Create the following presentation with the following slides using the beamer:

Slide 1: Title- Mean value Theorem; Author- ABC; Institute: XYZ University

Slide 2: Frame title- Statement

Let f: [a, b] → ℝ be a function such that

1. f is continuous on [a, b]

2. f is differentiable on (a, b)

Then ∃ atleast one point c ∈ (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Slide 3: Frame title- Examples

- sin x in [0, π]
- 1 + $1+\sqrt[3]{x-1}$ in [2,9]

Slide 4: Thank You.

(ii) Write a code in LaTeX to typeset the following :

Let $\vec{a} = < a_1, a_2, a_3 >$ and $\vec{b} = < b_1, b_2, b_3 >$ be vectors in R^3. Then the cross product is given by

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{\imath} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \hat{k}$$

Further, $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta$ where $\|\cdot\|$ and θ denote the length of the vector and angle between the vectors $\vec{a}$ and $\vec{b}$ respectively.

(iii) Find the errors and write the correct version of LaTeX source code (highlight your corrections in the answer). Also, write its output.

```
\begin{document}

\title{Maclaurin series for tan^{-1} x}

\author{ABC}

\maketitle

\begin{alignment}

\tan^{-1} x + c &=& \int \frac{1}{1+x^2}\, dx\\

&=& \int[1-x^2+x^4-\cdots]\, dx   (-1 < x < 1)\\

&=& \summation_{n=0}^{\infty} (-1)^n \frac{x^{2n+1} {2n+1}

\end{alignment}
```



```
Put x=0 and use tan^{-1} 0 = 0, we get

$$ \tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots (-1<x<1)$$

\end{document}
```

(iv) Make a parametric plot of lemniscate

$x = \frac{\cos t}{1+\sin^2 t}$ and $y = \frac{\sin t \cos t}{1+\sin^2 t}$ $\quad 0 \le t \le 360°$

Draw axes, label it and set unit-length of axes equal to 3 cm.

(v) Write an HTML code to generate the webpage under given instructions :

(a) Font face of the text should be "Arial"

(b) Text color of the main heading should be blue

(c) Make the text "Postgraduate" and "PhD" as a link by clicking the text the user reach to http://pgadmission.uod.ac.in and http://www.du.ac.in/index.php?page=ph.d. respectively.

[This question paper contains 4 printed pages.]

Your Roll No...............

G

Sr. No. of Question Paper : 4518

Unique Paper Code : 32351303

Name of the Paper : Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **Five** questions from each section.
4. **All** questions carry equal marks.

Section I

1. Find the following limits :

(i) $\lim\limits_{(x,y)\to(0,0)} \left(1+x^2+y^2\right)^{\frac{1}{x^2+y^2}}$

(ii) $\lim\limits_{(x,y)\to(0,0)} x \log\sqrt{\left(x^2+y^2\right)}$

2. Find an equation for each horizontal tangent-plane to the surface

$z = 5 - x^2 - y^2 + 4y$

3. The output at a certain factory is $Q = 150K^{\frac{2}{3}}L^{\frac{1}{3}}$ where K is the capital investment in units of \$1000, and L is the size of Labor force measured in worker-hours. The current capital investment is \$500,000 and 150 worker hours of Labor are used. Estimate the change in output that results when capital investment is increased by \$500 and Labor is decreased by 4 worker-hours.

4. Let $w = f(t)$ be a differentiable function of t where $t = (x^2 + y^2 + z^2)^{1/2}$. Show that

$(dw/dt)^2 = (\partial w/\partial x)^2 + (\partial w/\partial y)^2 + (\partial w/\partial z)^2.$

5. Let $f(x, y, z) = xyz$ and let $\hat{u}$ be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1, -1, 2)$ in the direction of $\hat{u}$.

6. Find the absolute extrema of the function $f(x, y) = e^{x^2-y^2}$ over the disk $x^2 + y^2 \le 1$.

Section II

1. Evaluate the double integral $\iint_D \frac{dA}{y^2+1}$ where D is triangle bounded by x=2y, y=-x and y=2.

2. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} y\sqrt{(x^2+y^2)}\, dy\, dx$ by converting to polar coordinates.

3. Find the volume of tetrahedron T bounded by plane $2x + y + 3z = 6$ and co-ordinate planes.

4. Use spherical co-ordinates to verify that volume of a half sphere of radius R is $\frac{2}{3}\pi R^3$.

5. Use cylindrical co-ordinates to compute the integral $\iiint_D z\left(x^2+y^2\right)^{-\frac{1}{2}} dx\, dy\, dz$ where D is the solid bounded above by the plane z=2 and below by the surface $2z = x^2 + y^2$.

6. Use a suitable change of variables to compute the double integral $\iint_D \left(\frac{x-y}{x+y}\right)^2 dy\, dx$, where D is the triangular region bounded by line $x + y = 1$ and co-ordinate axes.

Section III

1. Find the mass of a wire in the shape of curve C: $x = 3 \sin t$, $y = 3 \cos t$, $z = 2t$ for $0 \le t \le \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = z$.

2. Find the work done by force $\bar{F} = x\hat{i} + y\hat{j} + (xz - y)\hat{k}$ on an object moving along the curve C given by $R(t) = t^2\hat{i} + 2t\hat{j} + 4t^3\hat{k}$.

3. Use Green's theorem to find the work done by the force field $\bar{F}(x,y) = y^2\hat{i} + x^2\hat{j}$ when an object moves once counterclockwise around the circular path $x^2 + y^2 = 2$.

4. State and prove Green's Theorem.

5. Evaluate $\oint \left(2xy^2z\,dx + 2x^2yz\,dy + \left(x^2y^2 - 2z\right)dz\right)$ where C is the curve given by $x = \cos t$, $y = \sin t$, $z = \sin t$, $0 \le t \le 2\pi$ traversed in the direction of increasing t.

6. Use divergence theorem to evaluate $\iint_S \bar{F}.N\,ds$ where $\bar{F} = \left(x^5 + 10xy^2z^2\right)\hat{i} + \left(y^5 + 10yx^2z^2\right)\hat{j} + \left(z^5 + 10zy^2x^2\right)\hat{k}$ and S is closed hemisphere surface $z = \sqrt{1 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 1$ in x-y plane.

(1000)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4350 **G**

Unique Paper Code : 32351301

Name of the Paper : Theory of Real Functions

Name of the Course : **B.Sc. (H) Mathematics (LOCF)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) Let $A \subseteq \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A and $f: A \to \mathbb{R}$, then define limit of function f at c. Use $\epsilon - \delta$ definition to show that $\lim_{x \to 2} \frac{1}{1-x} = -1$. (6)

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$.

P.T.O.

Show that f has a limit at x = 0. Use sequential criterion to show that f does not have a limit at c if $c \neq 0$. (6)

(c) Show that $\lim_{x \to 0} \cos\left(\frac{1}{x^2}\right)$ does not exist in $\mathbb{R}$ but $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$. (6)

2. (a) Let $A \subseteq \mathbb{R}$, let $f: A \to \mathbb{R}$ and $g: A \to \mathbb{R}$ and $c \in \mathbb{R}$ be a cluster point of A. Show that if $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ then $\lim_{x \to c} (fg)(x) = LM$. (6)

(b) Evaluate the limit $\lim_{x \to 1+} \frac{x}{x-1}$. (6)

(c) Let $A \subseteq \mathbb{R}$, $f: A \to \mathbb{R}$ and $c \in \mathbb{R}$. Show that the following conditions are equivalent-

(i) f is continuous at c.

(ii) For every sequence (x_n) in A that converges to c, the sequence $(f(x_n))$ converges to f(c). (6)

3. (a) Let A, $B \subseteq \mathbb{R}$ and let $f: A \to \mathbb{R}$ and $g = B \to \mathbb{R}$ be functions such that $f(A) \subseteq B$. if f is continuous at a point $c \in A$ and g is continuous at $b \in B$, then show that the composition function $g \circ f: A \to \mathbb{R}$ is continuous at c. Also, show that the function $f(x) = \cos(1 + x^2)$ is continuous on $\mathbb{R}$. (7½)

(b) State and prove Maximum-Minimum Theorem for continuous functions on a closed and bounded interval. (7½)

(c) State Bolzano's Intermediate value theorem. Show that every polynomial of odd degree with real coefficients has at least one real root. (7½)

4. (a) Let $A \subseteq \mathbb{R}$ and $f: A \to \mathbb{R}$. Show that if f is continuous at $c \in A$ then $|f|$ is continuous at c. Is the converse true? Justify your answer. (6)

(b) Let I = [a, b] and $f: I \to \mathbb{R}$. Show that if f is continuous on I then it is uniformly continuous on I. (6)

(c) Show that f(x) = sin x is uniformly continuous on $\mathbb{R}$ and the function $g(x) = \sin\left(\frac{1}{x}\right)$, $x \neq 0$ is not uniformly continuous on $(0, \infty)$. (6)

5. (a) Let $I \subseteq \mathbb{R}$ be an interval, let $c \in I$, and let $f: I \to \mathbb{R}$ and $g: I \to \mathbb{R}$ be functions that are differentiable at c. Prove that the function fg is differentiable at c, and $(fg)' = f'(c)g(c) + f(c)g'(c)$. (6)

(b) Let $g: \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Show that g is differentiable for all $x \in \mathbb{R}$.

Also, show that the derivative g' is not continuous at x = 0. (6)

(c) Suppose that f is continuous on a closed interval I = [a, b], and that f has a derivative in the open interval (a, b). Prove that there exists at least one point c in (a, b) such that $f(b) - f(a) = f'(c)(b - a)$.

Suppose that $f: [0, 2] \to \mathbb{R}$ is continuous on [0,2] and differentiable on (0,2) and that f(0) = 0, f(2) = 1. Show that there exists $c_1 \in (0,2)$ such $f'(c_1) = 1/2$. (6)

6. (a) Find the points of relative extrema of the function $f(x) = 1 - (x - 1)^{2/3}$, for $0 \leq x \leq 2$. (6)

(b) Let I be an open interval and let $f: I \to \mathbb{R}$ has a second derivative on I. Then show that f is a convex function on I if and only if $f''(x) \geq 0$ for all $x \in I$. (6)

(c) Obtain Taylor's series expansion for the function $f(x) = \sin x$, $\forall x \leq \mathbb{R}$. (6)

(1000)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4915 **G**

Unique Paper Code : 42357501

Name of the Paper : Differential Equations

Name of the Course : **B.Sc. (Prog.) – DSE**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting **two** parts from each question.

3. Part of the questions to be attempted together.

4. If question paper has **Part- A/B/C** (write appropriate direction).

5. Use of non-programmable Scientific Calculator allowed.

P.T.O.

Attempt any **two** parts from each question.

1. (a) Solve $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$. (6.5)

   (b) Find the integrating factor and the solve the differential equation

   $(x^2 + y^2 + 1)dx - 2xydy = 0$ (6.5)

   (c) Solve $x \log x \frac{dy}{dx} + y = 2\log x$. (6.5)

2. (a) Solve $(D^2 + D)y = x^2 + 2x + 4$, where $D = \frac{d}{dx}$. (6)

   (b) Find a family of oblique trajectories that intersect the family of straight line $y = c\,x$ at an angle 45^0. (6)

   (c) Solve $xp^2 - 2yp + ax = 0$. (6)

3. (a) Using the method of variation of parameters, solve the differential equation

   $(x+4)\frac{dy}{dx} + 3y = 3$ (6.5)

   (b) Find the general solution of

   $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 2x \log x$$ (6.5)

   (c) Solve

   $tdx = (t - 2x)dt$

   $tdy = (tx + ty + 2x - t)dt$ (6.5)

4. (a) Solve

   $(D^3 - 3D^2 - 6D + 8)y = xe^{-3x}$, where $D = \frac{d}{dx}$. (6)

   (b) Solve

   $((D^2 - 1)y = x^2 \cos x$ (6)

   (c) Solve

   $(D^2 - 1)y = e^{-x} \sin e^{-x} + \cos e^{-x}$ (6)

5. (a) Form a partial differential equation corresponding to complete integral

   $x + y + z = f(x^2 + y^2 + z^2)$, where f is an arbitrary function. (6.5)

(b) Find the general solution of the linear partial differential equation

$$(y + zx)p - (yz + x)q = x^2 - y^2$$ (6.5)

(c) Find the integral surface of the partial differential equation

$$z(x + y)p + z(x - y)q = x^2 + y^2, \; y = 2x, \; z = 0$$ (6.5)

6. (a) Find the complete integral of the equation

$$(p + q)(px + qy) = 1$$ (6)

(b) Reduce the following partial differential equation into canonical form

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$$ (6)

(c) Solve

$$p \tan x + q \tan y = \tan z$$ (6)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4586 **G**

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the given **eight** questions are compulsory to attempt.

3. Do any **two** parts from each of the given eight questions.

4. Marks for each part are indicated on the right in brackets.

P.T.O.

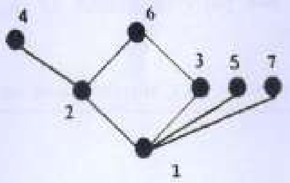
SECTION I

1. (a) Let $P = \{a, b, c, d, e, f, u, v\}$. Draw the Hasse diagram for the partially ordered set $(P; \leq)$, where the relations are given by:

$v < a,\ v < b,\ v < c,\ v < d,\ v < e,\ v < f,\ v < u,$
$a < c,\ a < d,\ a < e,\ a < f,\ a < u,$
$b < c,\ b < d,\ b < e,\ b < f,\ b < u,$
$c < d,\ c < e,\ c < f,\ c < u,$
$d < e,\ d < f,\ d < u,$
$e < u,\ f < u$ (2½)

(b) Give an example of a partially ordered set $(P; \leq)$ which is neither a chain nor an anti chain. Justify with suitable arguments as to why this partially ordered set $(P; \leq)$ is not a chain and not an antichain. (2½)

(c) Consider the diagram below of the ordered subset $P = \{1, 2, 3, 4, 5, 6, 7\}$ of $(N_0; \leq)$, where $(N_0; \leq)$ is the ordered set of non-negative integers ordered by relation $\leq$ on N_0 as: For $m, n \in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \in N_0$: $n=km$.



For the following subsets of P, find the following meet/join as indicated. Either specify the meet/join if it exists or indicate why it fails to exist.

(i) meet and join of subset {2,3,5}

(ii) meet and join of subset {2,3,6}

(iii) join of P (2½)

2. (a) Show that an order isomorphism for two ordered sets P and Q is a bijection, but the converse is not true. (3)

(b) Let P and Q be ordered sets. Prove that: $(a_1,b_1) \prec (a_2,b_2)$ in $P \times Q$ iff ($a_1 = a_2$ and $b_1 \prec b_2$) or ($a_1 \prec a_2$ and $b_1 = b_2$) (3)

(c) Let P, Q and R be ordered sets and let $\varphi: P \to Q$ and $\psi: Q \to R$ be order preserving maps. Then show that the composite map: $\psi o \varphi: P \to R$ given by:
$(\psi o \varphi)(x) = \psi((\varphi(x))$ for $x \in P$, is also an order preserving map. (3)

SECTION II

3. (a) Let $(L, \leq)$ be a lattice with respect to the order relation $\leq$. For the operations $\wedge$ and $\vee$ defined on L as:

$$x \wedge y = \inf(x, y),\ x \vee y = \sup(x, y)$$

show that $(L, \wedge, \vee)$ is an algebraic lattice, that is the associative laws, commutative laws, idempotency laws and absorption laws hold. (5)

(b) Define a lattice. Let $D_{24} = \{1,2,3,4,6,8,12,24\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '$\leq$' defined on D_{24} by $m \leq n$ iff m divides n, then show that D_{24} forms a lattice. (5)

(c) Let L_1 and L_2 be modular lattices. Prove that the product $L_1 \times L_2$ is a modular lattice. (5)

4. (a) Let L and K be lattices and $f: L \to K$ be a homomorphism. Then show that the following are equivalent

(i) f is order-preserving

(ii) $(\forall\ a, b \in L)$, $f(a \vee b) \geq f(a) \vee f(b)$

(5½)

(b) Let L be a lattice and let a, b, c Î L. Then show that :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ (5½)

(c) Define distributive lattice. Prove that homomorphic image of distributive lattice is distributive. (5½)

SECTION III

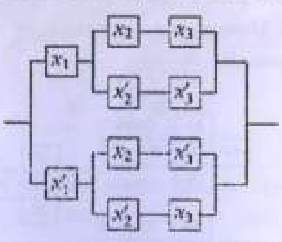
5. (a) Find the disjunctive normal form for (5½)

$$(x_1 + x_2 + x_3)(x_1 x_2 + x'_1 x_3)'$$

(b) Using Karnaugh diagram, simplify the expression

$$x_3(x_2 + x_4) + x_2 x'_4 + x'_2 x'_3 x_4$$ (5½)

(c) Find symbolic gate representation for (5½)



6. (a) Find the conjunctive normal form of $x_1(x_2+x_3)' + (x/x'_2\ x'_3)x_1$ in three variables. (5)

P.T.O.

(b) If B is the set of all positive divisors of 110, then show that (B, gcd, lcm) is a Boolean Algebra. (5)

(c) Find minimal form of the polynomial:

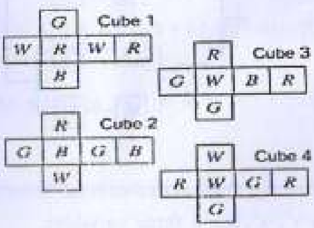
$$f = x'y + x'y'z + xy'z' + xy'z.$$

using Quine's McCluskey method. (5)

SECTION IV

7\. (a) What is the Three houses- Three Utilities Problem? How can it be formulated using graphs? Does this problem have a solution? (5½)

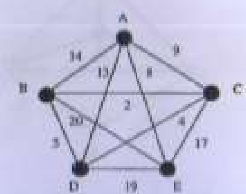
(b) Given four cubes as shown below (the cubes are cut along a few edges, then opened up and flattened) :



Find the solution, if it exists, for the game of "Instant Insanity" using the above four cubes, where the 6 faces of each of the four cubes have been coloured using four colours red(R), green(G), blue(B) and white(W). (5½)

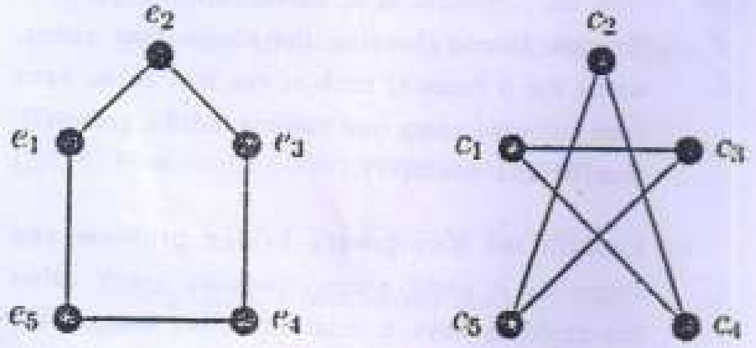
(c) Explain the Konigsberg bridge problem and formulate it using a corresponding graph. Does the problem have a solution? Give reasons for your answer. (5½)

8\. (a) Apply Improved Version of Dijkstra's Algorithm to find shortest distances from vertex A to all other vertices.

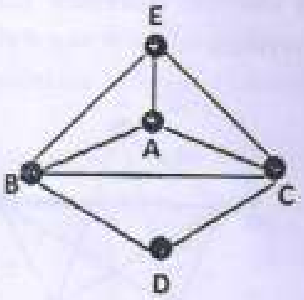


(5½)

(b) Consider the two graphs: the pentagon and the star as given below. Compute their adjacency matrices. Are they isomorphic to each other? If yes, exhibit an isomorphism between them. If not, then give suitable argument. (5½)



(c) (i) Define a Hamiltonian graph. Is the graph given below Hamiltonian? Explain.



(ii) Show how a Gray Code of length 2 can be constructed using a Hamiltonian cycle.

(5½)

[This question paper contains 4 printed pages.]

Your Roll No..............

G

Sr. No. of Question Paper : 4386

Unique Paper Code : 32351502

Name of the Paper : Group Theory – II

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Question No. **1** has been divided into **10** parts and each part is of **1.5** marks.
4. Each question from Q. Nos. **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each question.

1. State true (T) or false (F). Justify your answer in brief.

   (i) Let G be a finite group of order 147 then it has a subgroup of order 49.

   (ii) There is a simple group of order 102.

P.T.O.

(iii) Dihedral Group D_{12} (having 24 elements) is isomorphic to the symmetric group S_4.

(iv) The action $z \cdot a = z + a$ of the additive group of integers Z on itself is faithful.

(v) The external direct product $G \oplus H$ is cyclic if and only if groups G and H are cyclic.

(vi) Trivial action is always faithful.

(vii) The group of order 27 is abelian.

(viii) The external direct product $Z_2 \oplus Z_8$ is cyclic.

(ix) Every Sylow p-subgroup of a finite group has order some power of p.

(x) A p-group is a group with property that it has atleast one element of order p.

2. (a) Prove that for every positive integer n, Aut $(Z_n) \cong U(n)$.

(b) Define Automorphism Aut(G) of a group G and Inner Automorphism Inn(G) of the group G induced by an element 'a' of G. Prove that $Aut(Z_5)$ is isomorphic to U(5), where $U(5) = \{1, 2, 3, 4\}$ is group under the multiplication modulo 5.

(c) Define characteristic subgroup of G. Prove that every subgroup of a cyclic group is characteristic.

3. (a) Prove that the order of an element of a direct product of finite number of finite groups is the least common multiple of the orders of the components in the elements. Find the largest possible order of an element in $Z_{30} \oplus Z_{20}$.

(b) Prove that if a group G is the internal direct product of finite number of subgroups $H_1, H_2, \ldots H_n$ then G is isomorphic to $H_1 \oplus H_2 \oplus H_3 \ldots \oplus H_n$.

(c) Let G is an abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G.

4. (a) Show that the additive group R acts on x,y plane $R \times R$ by $r.(x,y)=(x+ry,y)$.

(b) Let G be a group acting on a non-empty set A. Define

(i) kernel of group action

(ii) Stabilizer of a in G, for $a \in A$

(iii) Prove that kernel is a normal subgroup of G.

(c) Define the permutation representation associated with action of a group on a set. Prove that the kernel of an action of group G on a set A is the same as the kernel of the corresponding permutation representation of the action.

5. (a) Let G be a group acting on a non-empty set A. If $a, b \in A$ and $b = g \cdot a$ for some $g \in G$. Prove that $G_b = g G_a g^{-1}$ where G_a is stabilizer of a in G. Deduce that if G acts transitively on A then kernel of action is $\cap_{g \in G} g G_a g^{-1}$.

(b) Define the action of a group G on itself by conjugation. Prove it is a group action. Also find the kernel of this action.

(c) If G is a group of order pq, where p and q are primes, $p < q$, and p does not divide q-1, then prove that G is cyclic.

6. (a) State the Class Equation for a finite group G. Find all the conjugacy classes for quaternion group Q_8 and also, compute their sizes. Hence or otherwise, verify the class equation for Q_8.

(b) Use Sylow theorems to determine if a group of order 105 is not simple.

(c) State and prove Embedding theorem and use it to prove that a group of order 112 is not simple.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4470 **G**

Unique Paper Code : 32357502

Name of the Paper : DSE-1 Mathematical Modelling and Graph Theory

Name of the Course : **B.Sc. (H) Mathematics – (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1\. (a) (i) Determine whether $x = 0$ is an ordinary point, a regular singular point or an irregular singular point of the differential equation

$$x^2y'' + (6 \sin x)y' + 6y = 0.$$

(ii) Find the Laplace transform of the function $f(t) = \sin 3t \cos 3t$

(iii) Find the inverse Laplace transform of the function $F(s) = \frac{9+s}{4-s^2}$. (6)

(b) Use Laplace transforms to solve the initial value problem :

$x'' + 6x' + 25x = 0;\ x(0) = 2,\ x'(0) = 3$ (6)

(c) Find two linearly independent Frobertius series solutions of (6)

$$4xy'' + 2y' + y = 0$$

(d) Find general solutions in powers of x of the differential equation. State the recurrence relation and the guaranteed radius of convergence.

$$5y'' - 2xy' + 10y = 0$$ (6)

2. (a) Explain Linear Congruence method for generating random numbers. Does this method have any drawback? Explain with the help of an example. (6)

(b) Use of Monte Carlo simulation to approximate the area under the curve $y = \cos x$ over the interval $-\pi/2 \le x \le \pi/2$, where $0 \le \cos x \le 2$. (6)

(c) Using algebraic analysis, solve the following:

Maximize : $x + 2y$

subject to $5x + 2y \le 10$

$2x + 3y \le 6$

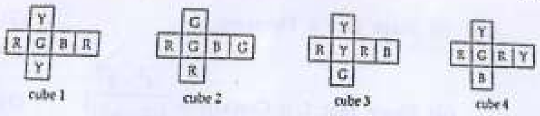
$x_1, x_2 \ge 0.$ (6)

(d) Consider a small harbor with unloading facilities for ships, where only one ship can be unloaded at any time. The unloading time required for a ship depends on the type and the amount of cargo. Below is given a situation with 5 ships:

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	20	30	15	120	25
Unload time	55	45	60	75	80

Draw the timeline diagram depicting clearly the situation for each ship. Also determine length of longest queue and total time in which docking facilities are idle. (6)

3. (a) Find the solution to the four-cubes problem for the following set of cubes. (6)



(b) Define semi-Eulerian trail. Prove that a bipartite graph with an odd number of vertices is not Hamiltonian. (6)

(c) Prove that there is no Knight's tour on a 7x7 Chessboard. (6)

(d) State Handshaking lemma. Use it to prove that in any graph, the number of vertices of odd degree is even. (6)

4. (a) Use the factorization :

$s^4 + 4a^4 = (s^2 - 2as + 2a^2)\ (s^2 + 2as + 2a^2)$

and apply inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s}{s^4 + 4a^4}\right\} = \frac{1}{2a^2}\sinh\ at\ \sin\ at$$ (7)

(b) Using Simplex method, solve the following linear programming problem :

Maximize $z = 5x_1 + 3x_2$

subject to $-3x_1 - 5x_2 \geq -15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$ (7)

(c) (i) State Ore's Theorem. (2)

(ii) Show that $L\{t\ Cos(kt)\} = \dfrac{s^2 - k^2}{\left(s^2 + k^2\right)^2}$. (5)

(d) Define Cube graphs. Write the number of vertices and number of edges in a cube graph Q_k. Draw Q_1, Q_2 and Q_3. (7)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4916 **G**

Unique Paper Code : 42357502-LOCF

Name of the Paper : Mechanics and Discrete Mathematics

Name of the Course : **B.Sc. Physical Sciences/ Mathematical Sciences (Part-III)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated.

P.T.O.

1. (a) (i) Particles of weights 1 lb, 2 lb, 3 lb, 4 lb are placed at the angular points of square, whose each side is of 'a' ft. in length. Find the mass centre of the system of particles. (4)

(ii) Find the potential energy of a particle attracted towards a fixed point by a force of magnitude $\frac{k^2}{r^n}$, r being the distance from the fixed point and k, n are constants. (3.5)

(b) A heavy particle rests on the top of smooth fixed sphere. If it is slightly displaced. Find the angular distance from the top at which it leaves the surface. Also find the velocity at that instant. (7.5)

(c) A particle moves in a plane with a constant speed. Prove that acceleration is perpendicular to its velocity. (7.5)

2. (a) A particle moves in a plane curve $r = ae^{\theta\cot\alpha}$, which is the equiangular spiral, and if the radius vector to the particle has a constant angular velocity, show that the resultant acceleration of the particle makes an angle 2α with the radius vector and is of magnitude $\frac{v^2}{r}$, where v is the speed of the particle. (7.5)

(b) A particle is performing an S.H.M. of period T about the centre O, and it passes through a position P, where OP = b, with velocity v in the direction OP. Prove that the time which elapses before its return to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$. (7.5)

(c) (i) A point moving in a straight line with SHM has velocities v_1 and v_2 when its distance from the centre are x_1 and x_2 respectively. Show that the period of motion is $2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$. (4)

(ii) At the end of three successive seconds the distances of a point moving in SHM from its extreme position measured in the same direction are 1, 5, 5. Show that the period of a complete oscillations is $\frac{2\pi}{\theta}$ where $\cos\theta = \frac{3}{5}$. (3.5)

P.T.O.

3. (a) Construct a precedence graph for the following program : (7.5)

S1: x := 0

S2: x := x + 1

S3: y := 2

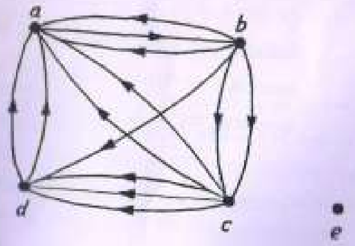
S4: z := y

S5: x := x + 2

S6: y := x + z

S7: z := 4

(b) Find the in-degree and out-degree of each vertex in the graph given below with directed edges.



Moreover, verify that

$$\sum_{v\in V} \deg^-(v) = \sum_{v\in V} \deg^+(v) = |E|$$

Here |E| is the number of edges. (7.5)

(c) What is Bipartite Graph. For which values of n, C_n and W_n are bipartite? (7.5)

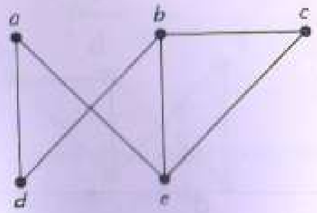
4. (a) Does each of these lists of vertices form a path in the following graph? Identify which paths are simple and which are circuits. What are the lengths of those that are paths? (7.5)

(i) a, e, b, c, b

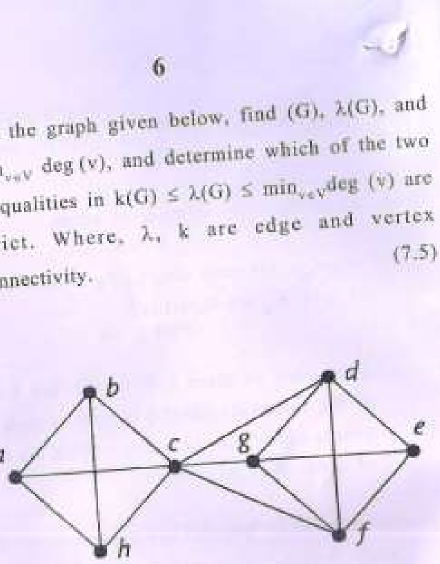
(ii) e, b, d, a, e, c, b, e

(iii) c, b, d, a, e, c, b, e

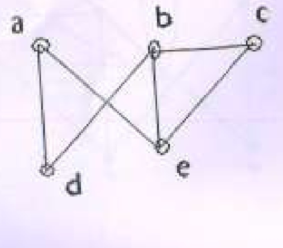
(iv) a, e, a, d, b, c, e, a



(b) For the graph given below, find (G), $\lambda(G)$, and $\min_{v \in V} \deg(v)$, and determine which of the two inequalities in $k(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$ are strict. Where, λ, k are edge and vertex connectivity. (7.5)

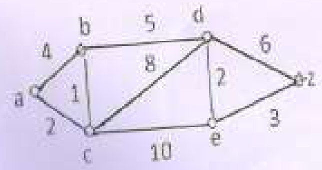


(c) How many paths of length four are there from a to c in the graph G? Identify all the paths. (7.5)



5. (a) What is the Konigsberg bridge problem? Write the graphical representation of this problem. Is it possible to cross all seven bridges in a continuous path without recrossing any bridge? Justify your answer. (7.5)

(b) Define weighted graph. Use Dijkstra's algorithm to find the length of a shortest path between a and z in the following weighted graph. (7.5)



(c) Define union of two simple graphs. Find the union of the graphs G_1 and G_2. (7.5)

[This question paper contains 4 printed pages.]

Your Roll No...............

G

Sr. No. of Question Paper : 4332

Unique Paper Code : 32351501

Name of the Paper : BMATH511 – Metric Spaces

Name of the Course : **B.Sc. (Hons) Mathematics (LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.

1. (a) Let (X, d) be a metric space. Show that (X, d^*) is a metric space where

$$d^*(x, y) = \min\{1, d(x,y)\}, \forall x, y \in X. \quad (6)$$

(b) (i) Let (X, d) be a metric space. Let $\langle x_n \rangle$ and $\langle y_n \rangle$ be sequences in X such that $\langle x_n \rangle$ converges to x and $\langle y_n \rangle$ converges to y. Prove that $d(x_n, y_n)$ converges to d(x, y). (2)

P.T.O.

(ii) Prove that if a Cauchy sequence of points in a metric space (X, d) contains a convergent subsequence, then the sequence converges to the same limit as the subsequence. (4)

(c) (i) Let X = ℕ, the set of natural numbers. Define $d(m, n) = \left|\frac{1}{m} - \frac{1}{n}\right|$; m, n ∈ X. Show that (X, d) is an incomplete metric space. (4)

(ii) Is the metric space (X, d) of the set X of rational numbers with usual metric d a complete metric space? Justify. (2)

2. (a) (i) Define an open set in a metric space (X, d). Show that every open ball in (X, d) is an open set. Is the converse true? Justify. (4)

(ii) Let S(x, r) be an open ball in a metric space (X, d). Let A be a subset of X such that diameter of A, d(A) < r and $S(x, r) \cap A \neq \emptyset$. Show that $A \subseteq S(x, 2r)$. (2)

(b) Let (X, d) be a metric space and A_1 and A_2 be subsets of X. Prove that $\overline{(A_1 \cup A_2)} = \overline{A_1} \cup \overline{A_2}$. Is the closure of the union of an arbitrary family of the subsets of X equal to the union of the closures of the members of the family? Justify. (6)

(c) Prove that a subspace of a complete metric space is complete if and only if it is closed. (6)

3. (a) Let (X, d_X) and (Y, d_Y) be two metric spaces. Show that a mapping f: X → Y is continuous if and only if for every subset F of Y, $(f^{-1}(F))^{\circ} \supseteq f^{-1}(F^{\circ})$. (6)

(b) (i) Let (X, d) be a metric space and A be a non-empty subset of X. Let f(x) = d(x, A) = inf {d(x, a), a ∈ A}, x ∈ X. Show that f is uniformly continuous over X. (4)

(ii) Is a continuous function over a metric space always uniformly continuous? Justify. (2)

(c) Let (X, d) be a metric space and $f: X \to \mathbb{R}^n$ be a function defined by $f(x) = (f_1(x), f_2(x) \ldots f_n(x))$, where $f_k: X \to \mathbb{R}$, $1 \leq k \leq n$ is a function. Show that f is continuous on X if and only if for each k, f_k is continuous on X. (6)

4. (a) Define homeomorphism between two metric spaces. Show that the image of a complete metric space under homeomorphism need not be complete. (6.5)

(b) Let d_1 and d_2 be two metrics on a non-empty set X. Show that d_1 and d_2 are equivalent if and only if the identity mapping I: $(X, d_1) \to (X, d_2)$ is a homeomorphism. (6.5)

(c) Let $T: X \to X$ be a contraction of a complete metric space (X, d). Show that T has a unique fixed point. (6.5)

5. (a) Show that the subset $A \subseteq \mathbb{R}^2$, where (6.5)
$A = \{(x, y) \in \mathbb{R}^2 : x^2 - y^2 \geq 9\}$ is disconnected.

(b) Let $I = [-1, 1]$ and let $f: I \to I$ be continuous, then show that there exists a point $c \in I$ such that $f(c) = c$. Discuss the result if $I = [-1, 1)$. (4+2.5)

(c) Let (X, d_X) be a connected metric space and f be a continuous mapping from (X, d_X) onto (Y, d_Y). Prove that (Y, d_Y) is also connected. Does there exist an onto continuous map $g: [0,1] \to [2, 3] \cup [4, 5]$? Justify your answer. (6.5)

6. (a) Let f be a continuous function from a compact metric space (X, d_X) to a metric space (Y, d_Y), then prove that f is uniformly continuous on X. (6.5)

(b) Let (X, d) be a metric space and Y be a compact subset of (X, d). Then prove that Y is closed and bounded. Give an example of a closed and bounded subset of a metric space which fails to be compact. (4+2.5)

(c) State finite intersection property. Show by using the finite intersection property that $(\mathbb{R}, d)$ with usual metric is not compact. (2+4.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4469 **G**

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All **six** questions are compulsory.

3. Attempt any **two** parts from each question.

4. Use of non-programmable scientific calculator is allowed.

P.T.O.

1. (a) Discuss the order of convergence of the Newton Raphson method. (6)

(b) Perform three iterations of the Bisection method in the interval (1, 2) to obtain root of the equation $x^3 - x - 1 = 0$. (6)

(c) Perform three iterations of the Secant method to obtain a root of the equation $x^2 - 7 = 0$ with initial approximations $x_0 = 2$, $x_1 = 3$. (6)

2. (a) Perform three iterations of False Position method to find the root of the equation $x^3 - 2 = 0$ in the interval (1, 2). (6.5)

(b) Find a root of the equation $x^3 - 5x + 1 = 0$ correct up to three places of decimal by the Newton's

Raphson method with $x_0 = 0$. In how many iterations does the solution converge? Also write down the order of convergence of the method used. (6.5)

(c) Explain the secant method to approximate a zero of a function and construct an algorithm to implement this method. (6.5)

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \quad 4 \quad 1]^T$. (6.5)

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$5x_1 + x_2 + 2x_3 = 10$$

$$-3x_1 + 9x_2 + 4x_3 = -14$$

$$x_1 + 2x_2 - 7x_3 = -33$$

Take the initial approximation as $X^{(0)} = (0,0,0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. (6.5)

4. (a) Construct the Lagrange form of the interpolating polynomial from the following data :

x	0	1	3
f(x)	1	3	55

(6)

(b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial.

x	0	1	2	3
y	-1	0	15	80

Hence, estimate the value of f(1.5). (6)

(c) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data :

x	−1	0	1	2
f(x)	3	−1	−3	1

(6)

5. (a) Derive second-order backward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}.$$ (6)

(b) Use the formula

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

to approximate the second derivative of the function $f(x) = e^x$ at $x_0 = 0$, taking h = 1, 0.1, 0.01 and 0.001. What is the order of approximation. (6)

(c) Approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 0$ using the first order forward difference formula taking h = ½, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the trapezoidal rule, approximate the value of the integral $\int_3^7 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Derive the Simpson's 1/3rd rule to approximate the integral of a function. (6.5)

(c) Apply the modified Euler method to approximate the solution of the initial value problem

$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 2, x(1) = 1$ taking the step size as $h = 0.5$. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4588 **G**

Unique Paper Code : 32357507

Name of the Paper : DSE – 2 : Probability Theory and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions selecting any **two** parts from each questions no.'s **1** to **6**.
3. Use of scientific calculator is permitted.

1. (i) If X has the probability density

$$f(x)=\begin{cases} ke^{-2x} & for\ x>0 \\ 0 & elsewhere \end{cases}$$

Find k and $P(0.5 \le X \le 1)$. Also find the distribution function of the random variable X and use it to reevaluate $P(0.5 \le X \le 1)$. (6)

(ii) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2)=\begin{cases} x_1+x_2 & \text{if } 0<x_1<1,\ 0<x_2<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal pdf of X_1 and X_2 and compute $P(X_1 + X_2 \le 1)$. (6)

(iii) Let X be a continuous random variable with pdf $f(x) = ke^{-kx}$, $0 \le x < \infty$. Find E(X), E(X^2), Var(X) and the cumulative distribution function. (6)

2. (i) If a random variable X has a discrete uniform distribution $f(x) = \frac{1}{k}$ for $x = 1,2,3...k$, then find the mean and variance. (6)

(ii) Let the random variables X and Y have the joint pdf

$$f(x,\ y)=\begin{cases} e^{-y} & \text{if } 0<x<y<\infty \\ 0, & \text{elsewhere} \end{cases}$$

Find the moment generating function of the joint distribution. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x,\ y)=\begin{cases} 2 & \text{if } 0<x<y<1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the conditional pdf of X given Y = y and the conditional pdf of Y given X = x. (6)

3. (i) If X is a Poisson distributed random variable with parameter λ then prove that

$$\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right] \text{ for } r = 1, 2, 3, \ldots \qquad (6)$$

(ii) If the probability is 0.60 that a girl child exposed to a certain contagious disease will catch it, what is the probability that the eleventh girl child exposed to the disease will be the fifth to catch it? (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} 6y & \text{if } 0 < y < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Define $Z = \frac{2X}{3}$, find the mean and variance of Z. (6)

4. (i) If a random variable X has a beta distribution then show that its mean and variance are given by :

$$\mu = \frac{\alpha}{\alpha+\beta} \text{ and } \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \qquad (6.5)$$

(ii) If the exponent of e of a bivariate normal density of random variables X and Y is

$$\frac{-1}{102}[(x+2)^2 - 2.8(x+2)(y-1) + 4(y-1)^2]$$

then find mean of X, mean of Y, standard deviation of X, standard deviation of Y and the correlation coefficient of X and Y. (6.5)

(iii) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} 8x_1x_2 & \text{if } 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Are X_1 and X_2 independent? (6.5)

5. (i) Suppose the joint moment generating function, $M(t_1, t_2)$, exists for the random variables X and Y. Then X and Y are independent if and only if

$M(t_1, t_2) = M(t_1, 0)M(0, t_2)$. (6.5)

(ii) If the probability density of X is given by

$$f(x) = \begin{cases} 630x^4(1-x)^4, & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that it will take on a value within two standard deviations of the mean and compare this probability with the lower bound provided by Chebyshev's theorem. (6.5)

(iii) The mean height of 500 students is 151 cm and the standard deviation is 15 cm. assuming that the heights are normally distributed, find how many students have heights between 120 and 155 cm? (6.5)

6. (i) Calculate the correlation coefficient for the following heights (in inches) of father's (X) and their son's (Y): (6.5)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(ii) If $X_1, X_2, X_3, \ldots\ldots X_n$ constitute a random sample from an infinite population with mean μ, the variance σ^2 and the moment generating function $M_X(t)$ then show that, the limiting distribution of $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ as $n \to \infty$ is the standard normal distribution. (6.5)

(iii) If X is a random variable that takes only nonnegative values, then Show that for any value $a > 0$,

$$P\{X \geq a\} \leq \frac{E[X]}{a} \qquad (6.5)$$